Radiation Energy Receiver for Solar Propulsion Systems

D. F. G. Rault*
McGill University, Montreal, Canada

The concept of remotely heating a rocket propellant with a high-intensity radiant energy flux is especially attractive due to its high specific impulse and large payload mass capabilities. In this paper, a radiation receiver-thruster which is especially suited to the particular thermodynamic and spectral characteristics of highly concentrated solar energy is proposed. In this receiver, radiant energy is volumetrically absorbed within a hydrogen gas seeded with alkali metal vapors. The alkali atoms and molecules absorb the radiant flux and subsequently transfer their internal excitation to hydrogen molecules through collisional quenching. It is shown that such a radiation receiver would outperform a blackbody cavity type receiver in both efficiency and maximum operating temperatures. A solar rocket equipped with such a receiver-thruster would deliver thrusts of several hundred Newtons at a specific impulse of 1000 seconds.

Nomenclature

B_ν = black body Planck function
C_p = heat capacity of gas mixture
C_{pe}, C_{pf} = heat capacity of hydrogen-equilibrium-frozen
D/f = aperture ration of solar collector
E_2 = integro exponential function of order two
f = solar collector focal length
F = solar rocket thrust
g = gravity acceleration
$I_{\nu 0}$ = spectral intensity incident on solar collector
k = working fluid thermal conductivity
K_p = equilibrium constant between atoms and dimers
$\dot{m}$ = mass flow rate
M_A, M_H = molecular mass of alkali vapor and hydrogen gas
M_0, M_f = initial, final mass of solar rocket
MR = M_0/M_f
N_u = Nusselt number
P_A, P_D = partial pressure of monatomic and diatomic alkalis
S = cross-sectional area of absorber cavity
S_ν = $\int_0^x \alpha_\nu \mathrm{d}x$
T_i, T_t = gas temperature at inlet and nozzle throat
T_0, T_e = maximum gas temperature and nozzle exit temperature
T_w, T_c = wall temperature, cryogenic hydrogen temperature
u, u_e = gas axial velocity at x and at exit
x = axial coordinate
α = alkali weight concentration
α_ν = absorption coefficient
ΔV = velocity increment from LEO to GEO
η_1 = solar collector efficiency
η_2 = energy concentrated within focal disc/energy reflected by solar collector (Ref. 21).
η_3 = radiation receiver efficiency
ν_1, ν_2 = lower and upper frequency of absorption region
ρ = gas density
τ_q, τ_{alk} = characteristic time for quenching, emission

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*Visiting Associate Professor. Currently Research Scientist, Science Applications International Corporation. Member AIAA.

I. Introduction

FUTURE orbital and planetary missions with large velocity increments will require the development of high specific impulse rockets with large initial to final mass ratios. Radiation-heated propulsion systems have shown promising features and have been extensively studied and reviewed in the literature.[1-8] Laser- and solar energy-propelled rockets could utilize the high thermodynamic potential of coherent radiant energy and concentrated solar energy and deliver substantial thrusts at high specific impulses. In this paper, the potential of solar propulsion systems is reviewed and a high-efficiency, radiation-heated rocket engine, especially suited to the particular thermodynamic and spectral characteristics of highly concentrated solar energy, is proposed.

The first part of the paper reviews and discusses the various schemes which have been proposed in the literature to couple a high-intensity light beam to a space rocket propellant. Section III describes a high-efficiency radiation receiver for solar energy application whose basic concept was developed and main characteristics analyzed during an investigation of high-efficiency conversion of solar energy in space.[9,10] In this receiver, alkali metal vapors, which strongly absorb visible radiation over a wide spectral range, are used to couple the solar energy radiation to the working fluid. The potential of this receiver for space propulsion is then examined and a receiver-thruster combination is proposed and analyzed. Specific impulses of 1000 s are shown to be attainable with a propellant composed of a mixture of alkali metal vapor and hydrogen.

In Sec. V, the overall performance of a solar rocket equipped with an alkali/hydrogen receiver thruster is analyzed for a typical Low Earth Orbit-to-Geosynchronous Earth Orbit (LEO-to-GEO) mission originating from a space shuttle cargo bay. It is shown that thrusts of a few hundred Newtons (1 N = 0.22 lb.) and a mission time of a few days can be achieved with a payload mass ranging from 40 to 50% of the initial mass.

II. Coupling of Radiation Energy to a Rocket Propellant

The concept of remotely heating a rocket propellant with a high-intensity light beam has been studied mainly using high-power infrared lasers as light sources.[1-7] Jones and Keefer[1] and Hofer[2] have described and studied a laser-heated rocket engine in which the high-power laser beam is absorbed via inverse bremsstrahlung within a 15,000 K hydrogen plasma (Fig. 1a). The plasma subsequently transfers its energy through convection and radiation to a hydrogen propellant, which is then expanded through a conventional rocket nozzle. Several research investigations were sponsored to examine the physics of the laser-supported plasma, especially the difficult problem of plasma stability, and to evaluate the reradiation losses and the cooling requirements of the absorption chamber.

Huberman et al.[3] and Hofer[2] have suggested and studied other methods for coupling the laser energy to the propellant. These alternative methods are simpler in that they do not necessitate the sustaining and control of a plasma, but they are confined to lower temperatures and, hence, to lower specific impulse applications. As shown by Huberman, infrared laser energy can be absorbed by the rotational-vibrational bands of diatomic and polyatomic molecules (Fig. 1b). Molecular absorbers, however, generally have a high molecular mass and dissociate at elevated temperatures. Moreover, most of the absorbed energy is stored in the internal degrees of freedom of the molecules and propellant heating is slow and inefficient if the relaxation of excited molecules is slow.

The absorption of the laser beam by a cloud of solid particles or aerosols (Fig. 1c)[2,3] has the advantage of being broadband. Particulates would need to be uniformly distributed and extremely small to ensure an efficient transfer of their excess energy to the gas and minimize the scattering of the light beam to the thruster walls. Moreover, solid particles would tend to erode the thruster surfaces or deposit onto the chamber window. Means to minimize these effects would have to be developed.

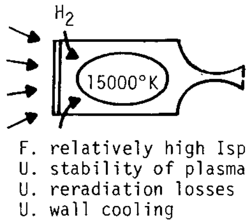

Fig. 1a Coupling of a high-intensity radiant flux to a rocket propellant, plasma absorber. (F refers to favorable, U to unfavorable features).

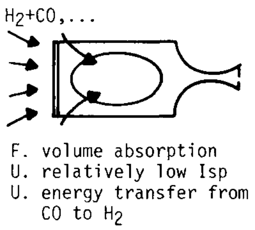

Fig. 1b Coupling of a high-intensity radiant flux to a rocket propellant, gaseous volume absorber. (F refers to favorable, U to unfavorable features).

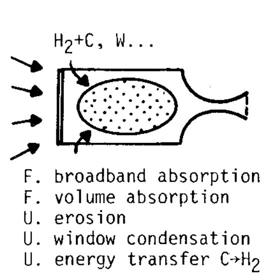

Fig. 1c Coupling of a high-intensity radiant flux to a rocket propellant, particulate volume absorber.(F refers to favorable, U to unfavorable features).

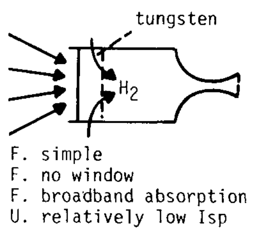

Fig. 1d Coupling of a high-intensity radiant flux to a rocket propellant, surface absorber. (F refers to favorable, U to unfavorable features).

The absorption of the light beam on a solid heat exchanger surface (Fig. 1d) or within a blackbody-like cavity, would be the simplest method for transferring radiant energy to a fluid. It is to date the only method which has been used to transfer solar energy to a fluid on a significant scale.[11] The mechanical and corrosive stresses within the structural walls, however, would become excessive at elevated temperatures and a space propulsion system equipped with such a receiver would be limited in performance and have a relatively low specific impulse.

III. High-Efficiency Solar Energy Radiation Receiver Basic Concept

The radiation receiver which is proposed in this paper is basically a gaseous volume receiver (as depicted in Fig. 1b). Its main features are shown in Fig. 2. The concentrated sunlight is transmitted through a transparent window and is absorbed within a volume of alkali metal vapors, selected as working fluids due to their strong and wide spectral band absorption in the visible spectrum.[10,12] The main absorption mechanisms in alkali vapors are transitions between electronic levels of atoms and dimers. The loosely bound dimers contribute to most of the absorption but readily dissociate at elevated temperatures.[10,13] Alkai metal absorbers would therefore be confined to moderately high temperatures ($T<5000$ K), but would be perfectly suited for solar applications. Other absorption mechanisms such as inverse bremsstrahlung,[9] photoionization,[14] and two-step photoionization[15] could become important at elevated temperatures and extend the operating range of this class of absorbers.

Mattick et al.[9] investigated the alkali metal receiver for the conversion of solar energy in space and showed that it could outperform a blackbody energy-exchanger type receiver at all temperatures. Figure 3 displays the collection effeciency of both types of receivers for an input solar beam of 1 KW/cm^2. (Conduction and convection losses to the walls were neglected and the receiver inside walls were assumed to be perfectly

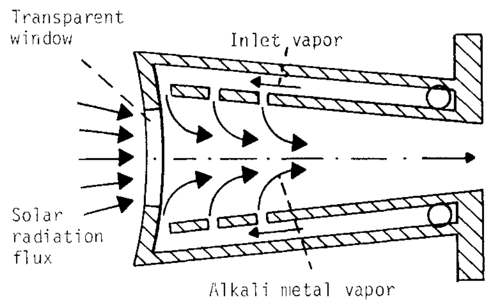

Fig. 2 Alkali metal volume absorber (Refs. 9 and 10).

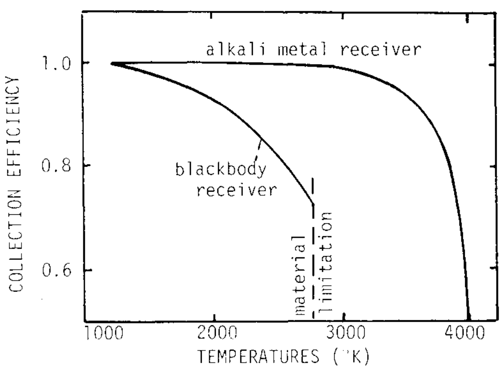

Fig. 3 Performance of an alkali metal receiver and a blackbody receiver.

Table 1 Alkali metal properties

	Lithium	Sodium	Potassium	Cesium
Molecular weight	7	23	39	133
Melting point, °C	179	98	64	29
Ionization potential, eV	5.39	5.14	4.34	3.89
P atom lifetime, ns	27.2	16.1	27.8	30.5
Dimer $A\Sigma$ dissociation energy, eV	1.25	1.01	0.68	0.63
Dimer $B\pi$ dissociation energy, eV	0.46	0.35	0.22	—

Table 2 Coupling between a visible light beam and a gas composed of alkali vapor and hydrogen

Absorption or radiant energy by alkali atoms and molecules

$$A,\quad A_2 + h\nu \rightarrow A^*,\quad A_2^*$$

Collisional quenching of alkali atoms by diatomic molecules:

$$A^* + H_2 \rightarrow A + H_2^V + \Delta E$$

Relaxation of optically excited alkali molecules:

$$AA^* + H_2 \rightarrow A + A^* + H_2 \rightarrow 2A, A_2 + H_2^V + \Delta E$$

Relaxation of vibrationally excited hydrogen molecules:

$$H_2^V + H_2 \rightarrow 2H_2 + \Delta E$$

Note: The superscripts *, V refer to an electronic and vibrational excess energy respectively. "A" represents an alkali atom. ΔE is an increment of thermal energy.

reflecting). In a blackbody receiver, the reradiation losses increase as σT^4, whereas in a gaseous volume absorber, the reradiation emitted by the hot gases downstream is partially reabsorbed by the cooler gases entering the receiver near the front window, and reradiation losses are much lower. With its low reradiation characteristics and its ability to sustain very high temperatures (the walls can be cooled by regenerative counterflow, transpiration, or film cooling), the alkali metal receiver can be used to convert solar energy into thermal energy at high efficiency and over a wide range of temperatures.

Alkali Metal Receiver for Space Propulsion

Due to their large molecular weight (Table 1), alkali metals alone cannot be considered as propellants for thermal space propulsion systems. Hydrogen, which is usually selected as a propellant in these systems, is virtually transparent in the visible and infrared, and cannot be considered in solar gaseous volume absorbers. A propellant composed of a mixture of alkali metal vapors and hydrogen could, however, feature a low molecular weight and desirable optical properties. Figure 4 shows the molecular weight of such a propellant as a function of the weight concentration α of alkali metal and the total pressure P. At elevated temperatures, an average molecular mass of less than 2 can be achieved as hydrogen molecules dissociate.

The absorption strength of a hydrogen-alkali propellant is illustrated on Fig. 5 (see Appendix for details). A propellant composed of 10% potassium and 90% hydrogen (by weight) at a chamber pressure of 100 atm would absorb radiant energy effectively in the spectral range 0.6 to 1.0 μm within a distance of 1 m at temperatures up to 5000 K. If sodium is substituted for potassium, the propellant would predominantly absorb in the visible region from 0.44 to 0.8 μm. At lower chamber pressures, higher alkali concentrations are required to maintain high absorptivity level, and the average molecular mass of the propellant would be higher.

The injection of hydrogen within an alkali metal radiation receiver could further improve the overall performance of the basic receiver. As shown in Fig. 6, hydrogen reacts with alkali metals at high temperatures and pressures to form hydrides.[16] The diatomic hydrides absorb strongly in the shorter wavelengths[17] and would, therefore, increase the absorption spectral range of the propellant. Furthermore, diatomic molecules such as nitrogen and hydrogen are known to quench the excess electronic energy of excited alkali atoms effectively[18-20] and could significantly increase the rates of conversion of the internal energy of optically excited alkali atoms and molecules into thermal energy via the processes depicted in Table 2. In Fig. 7, the characteristic time τ_Q for the quenching of potassium and sodium atoms by hydrogen molecules is compared to the radiative lifetime τ_{alk} of the excited alkali atoms (in the resonance P level) for several stagnation pressures and temperatures. At pressures above 10 atm, excited alkali atoms would readily transfer their excess electronic energy to hydrogen molecules and barely reradiate. At low pressures, hydrogen molecules dissociate excessively at elevated temperatures and the quenching of alkali atoms would be less efficient. The excess vibrational energy thus

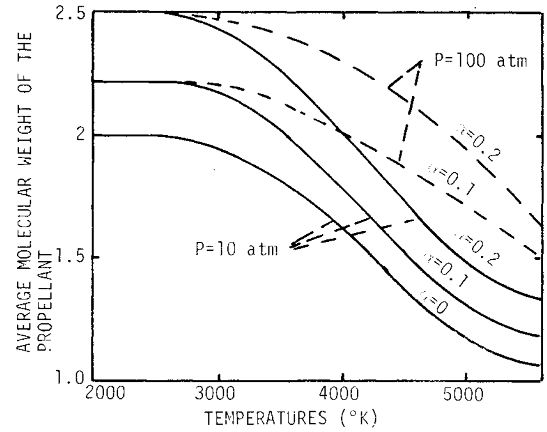

Fig. 4 Average molecular weight of a propellant composed of hydrogen and heavy alkali metal (potassium, sodium).

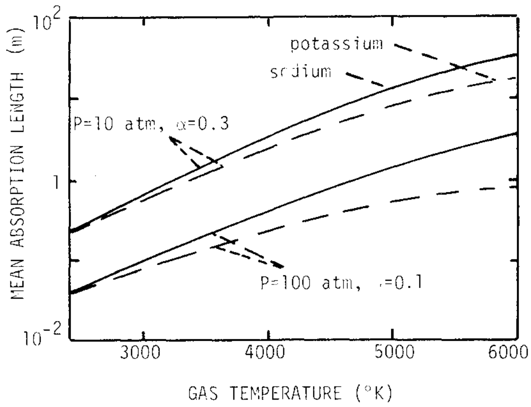

Fig. 5 Mean absorption length of hydrogen-potassium (between 0.6 and 1 μm) and hydrogen-sodium (between 0.44 and 0.8 μm).

acquired by hydrogen molecules is quickly released through collisions at elevated pressures and temperatures. The rates of relaxation of excited alkali molecules are also enhanced by the presence of high-pressure hydrogen. Excited alkali molecules in their lower ($A\Sigma$) and upper ($B\pi$) states are weakly bound (Table 1) and readily dissociate upon collisions with neighboring gas particles. Hydrogen molecules subsequently quench the resulting excited alkali atoms.

The relaxation of optically excited alkali atoms and molecules in an alkali-hydrogen receiver would, therefore, be very fast. The solar energy absorbed by the gas can be considered to be instantaneously converted into thermal energy. The working gas in the receiver can therefore be assumed to remain in thermodynamic equilibrium.

The selection of the alkali metal dopant depends not only on its absorption characteristics but also on its thermodynamic properties. The alkali metal must be injected into the radiation receiver chamber as a dry vapor to ensure maximum collection efficiency and prevent light scattering by liquid droplets. Boiling of sodium, potassium, and cesium could be achieved readily through the cooling of very hot sections of the thruster, such as the nozzle throat. Lithium, however, has a very high boiling point, and, as shown in Fig. 6, strongly reacts with hydrogen. Moreover, its radiation absorption characteristics are not well known. Consequently, in this paper lithium was not considered, in spite of its low molecular weight.

Performance of Alkali/Hydrogen Radiation Receiver

The fundamental characteristics and performance of the alkali/hydrogen radiation receiver were determined using a quasi-one-dimensional analytical model (see Appendix). Figure 8 shows the collection efficiency of a 50-cm-diam receiver operating with a working fluid composed of 90% hydrogen and 10% alkali with an equimolar concentration of potassium and sodium. The inlet gas temperature is 1200 K and the chamber pressure is 100 atm. The geometric aspect ratio of the receiver cavity (length to diameter) is 1. The solar mirror was assumed to be a perfect parabolic concentrator (no surface and tracking errors) with an optical efficiency $\eta_t = 80\%$ and a focal length f such that the sun's image diameter is equal to the receiver diameter ($f = 54$m).[21,22] The propellant is assumed to be transparent outside the main spectral absorption region which extends from 0.44 to 1.0 μm. Absorption by inverse bremsstrahlung (alkali atoms and ions, hydrogen atoms, ions, molecules), photoionization, photodetachment (from H^-),[23,24] and absorption by hydride and Na K molecules were not accounted for due to the lack of data on their relative strengths and on the relaxation kinetics of the resulting excited particles.

The performance of the alkali vapor receiver is compared in Fig. 8 to the performance of a blackbody type receiver illuminated by a filtered sunlight beam with a spectral distribution extending from 0.44 to 1.0 μm. (This spectral range accounts for 53% of the total sunlight energy). Due to reradiation trapping, for equal concentrator aperture ratio D/f the collection efficiency of an alkali metal receiver exceeds the efficiency of a blackbody receiver. Turbulent convective heat loss to the side walls was assumed to be the predominant loss mechanism in the radiation receiver as the side walls were assumed to be perfectly reflecting. For a mirror aperture ration $D/f = 2$, the convective heat loss to the receiver walls increases from 5 to 8% while the reradiation loss through the front window increases from 4 to 40% as the temperature is increased from 3000 to 3900 K.

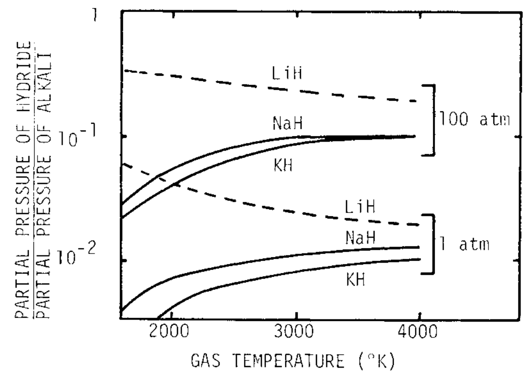

Fig. 6 Equilibrium concentration of hydrides in a mixture of hydrogen and alkali metal at pressures of 1100 atm and alkali concentration $\alpha = 0.1$ (Ref. 16).

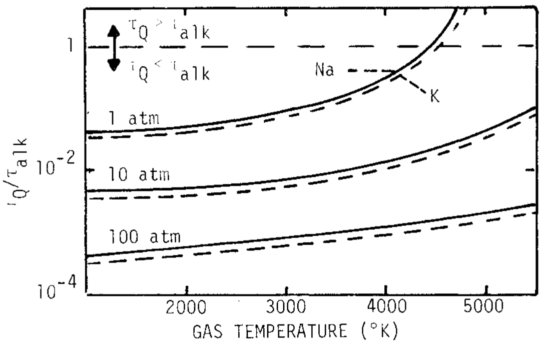

Fig. 7 Characteristic time for the quenching of excited alkali atoms by hydrogen (Refs. 18-20).

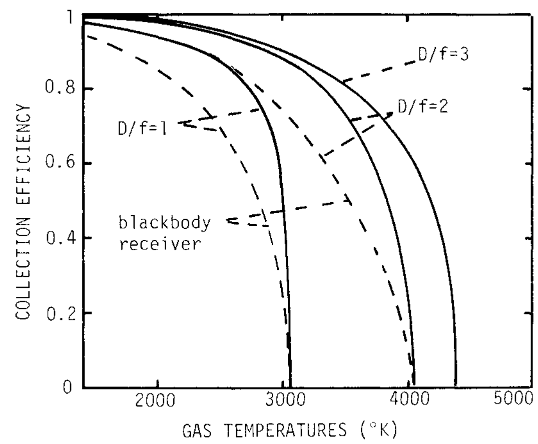

Fig. 8 Performance of an alkali metal solar receiver.

IV. Solar Rocket Configuration and Performance Radiation Receiver-Thruster Configuration

Figure 9 illustrates the main features of a radiation receiver-thruster which could be used for solar propulsion. The high-intensity sunlight is transmitted through a sapphire window which is face-cooled with pure hydrogen. The premixed alkali-hydrogen propellant is preheated (up to at least 1200 K to maintain a dry alkali vapor) in a front flat disc absorber and along the chamber walls (regeneration) before entering the receiver chamber. The blackcoated flat disc absorber collects the relatively low-concentration solar energy which is reflected by the solar concentrator onto the periphery of the radiation receiver. The hot gases are contained away from the structural walls (through film cooling) as they flow through the absorption chamber and a convergent-divergent nozzle.

Specific Impulse

The specific impulses which can be reach with an alkali/hydrogen radiation receiver-thruster are shown in Fig. 10, for stagnation temperatures up to 5500 K and chamber pressures of 10 and 100 atm (see Appendix). The pressure ratio across the exit nozzle is assumed to be 10^4. Specific impulses of about 1000 s can be obtained at a chamber temperature of 4000 K for alkali weight concentration factors α less than 0.3. Large chamber pressures yield the best performance since lower alkaki metal weight concentrations are needed to ensure adequate absorption of the light beam by the propellant. Lower pressures might be required, however, to

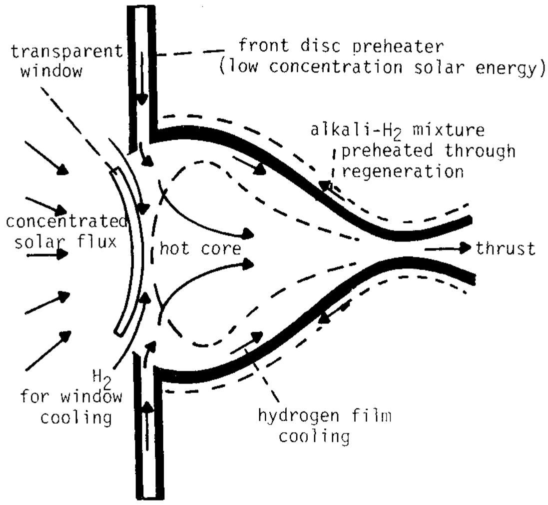

Fig. 9 Schematic diagram of a solar radiation receiver thruster. Aperture ratio $D/f < 2$.

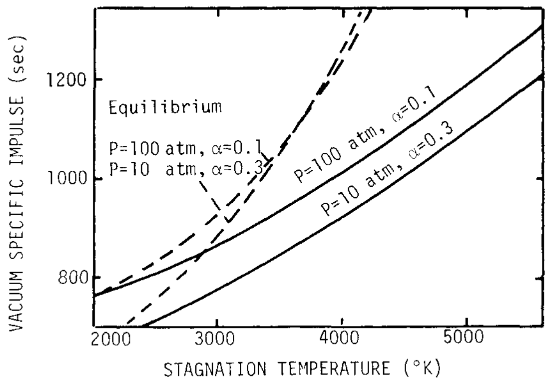

Fig. 10 Specific impulse of a hydrogen-potassium propellant.

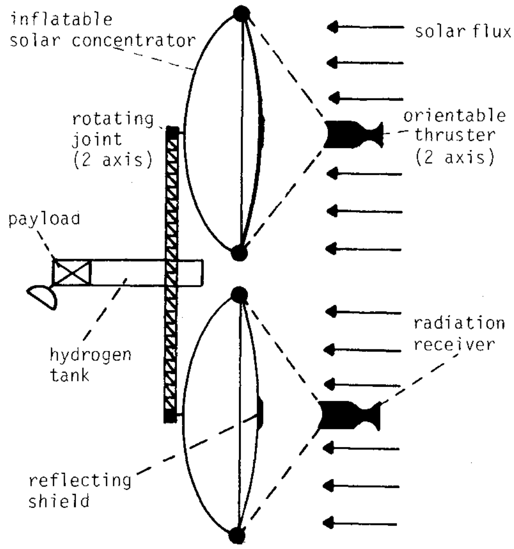

Fig. 11 Solar rocket configuration.

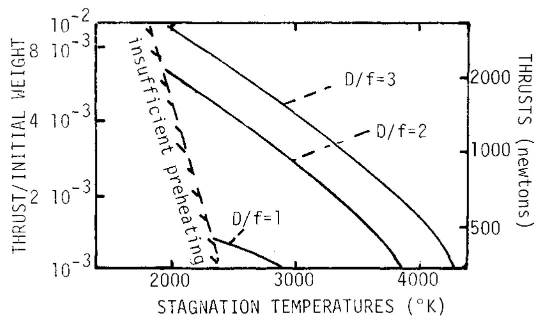

Fig. 12 Thrust of a solar rocket with a 50-cm radiation receiver.

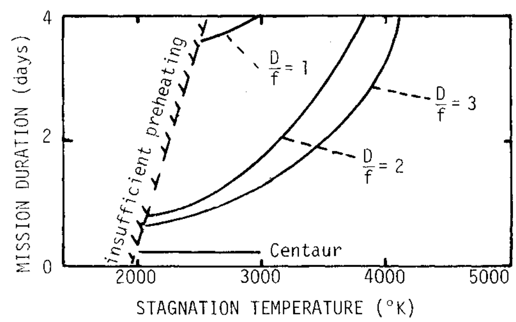

Fig. 13 Minimum duration of a typical mission from LEO to GEO with a solar rocket (continuous spiral burn).

permit efficient cooling of the nozzle throat and minimize the mechanical stresses borne by the receiver window. The performances shown in Fig. 10 were derived assuming frozen flow conditions (no recombination and no vibrational relaxation of hydrogen) in the diverging section of the exit nozzle. The kinetics of atomic hydrogen recombination are relatively slow,[25] but could be enhanced through the addition of catalysts.[26] The presence of alkali atoms in the propellant should be investigated for its effect on recombination kinetics.

Solar Rocket Configuration

The basic configuration of a solar rocket is shown in Fig. 11. The radiation receiver-thruster is gimbal-mounted at the focal point of an on-axis inflatable solar concentrator which constantly tracks the sun. Inflatable solar mirrors were selected due to their high surface quality, low specific mass, and deployment characteristics.[8] The gas leaks resulting from defective seals or micrometeorite impacts were determined by Etheridge[8] to be sufficiently small to require only a small amount of makeup gas during a typical mission a few days long. As shown in Fig. 11, the solar rocket could be composed of several concentrator-thruster units for stability control and trajectory changes. The optimal design of a solar rocket would have to take into account the specific transfer orbit mechanics and maneuver modes (continuous spiral burn, multiple "impulsive" burns,[8] etc.).

Solar Rocket Performance

The performance of a solar rocket equipped with one alkali/hydrogen radiation receiver-thruster was evaluated for a typical LEO-to-GEO mission. The mission was considered as orginating from a space shuttle cargo bay and to have an initial mass of 29,500 kg and an initial volume of hydrogen occupying no more than 70% of the cargo bay. The receiver chamber is 50 cm in diameter and the solar mirror focal length is $f = 54$m. The propellant is composed of 90% hydrogen and 10% alkali metal (with an equimolar concentration of sodium and potassium) at 100 atmospheres. Figures 12 and 13 show the thrust of the solar rocket and the mission duration as a function of stagnation temperatures and solar mirror aperture ratios, respectively (see Appendix). It can be seen that a solar rocket equipped with an alkali vapor radiation receiver would deliver thrusts in the range of 400 to 2000 N and reach the geostationary orbit in 1 to 4 D. For comparison, a chemical Centaur rocket with a thrust of 130 N and $Isp = 475$ s would travel from LEO to GEO in 5 h, while a typical mercury ion thruster would complete the orbit transfer in 180 D at thrust levels of 9 N.[8] Assuming a structural mass of 10% of the initial mass, the payload mass fraction of a solar rocket would be 45% in the range 3500-4000 K, that is, nearly twice that of a chemical rocket and only 30% less than that of a mercury ion thruster with a specific impulse of 3000 s. Larger payload weights could be achieved at higher stagnation temperatures at the expense of lower thrusts (due to lower collection efficiency), and larger mission durations. The results shown in Figs. 12 and 13 were derived assuming a continous burn spiral transfer orbit. Etheridge[8] has shown that the payload mass capability of a low-thrust rocket could be improved with a multiple impulse orbit transfer (burns at apogee and perigee). The dotted limit curves in Figs. 12 and 13 indicate that the propellant preheat in the front disc absorber and regeneration (assumed 80% efficient) is insufficient at high mass flow rates, and the alkali vapor entering the radiation receiver cannot be maintained dry.

The exit nozzle of a solar rocket operating at 100 atm would be fairly compact. For a 50-cm receiver and stagnation temperatures ranging from 2000 to 5000 K, the nozzle throat diameter varies from 1.5 to 0.5 cm, and the nozzle exit diameter varies from 20 to 10 cm.

V. Conclusions

High-concentration solar energy radiation can generate propellant gas temperatures which are higher than possible in chemical propulsion devices, and high specific impulses can be attained when using a low-molecular-weight propellant. To exploit the potential of a solar rocket fully, however, the energy transfer from the solar beam to the propellant must be accomplished at the highest efficiency, which ensures high thrusts and short mission duration, and at the highest temperatures, which ensures high specific impulses and large payload mass. A high-efficiency, high-temperature solar radiation receiver-thruster has been proposed and analyzed, in which a propellant composed of a mixture of hydrogen and alkali metal vapor is volumetrically heated by radiation absorption. The radiant energy is quickly transferred to the hydrogen gas via the electronic energy levels of the strongly absorbing alkali metal vapors. The alkali/hydrogen receiver-thruster was shown to be capable of delivering thrusts of several hundred N at specific impulses of about 1000 s. Such thrust levels correpond to the acceleration range 0.01-0.1 g (for an initial mass of 29,500 kg) which would be optimum for large space structures and other frail payloads, and would correspond to a trip time from LEO to GEO of a few D.

The promising features of the solar-heated thruster presented in this paper and its potential for future space propulsion systems warrant further investigation of its characteristics and applications. Hydrogen was selected as the propellant due to its low molecular weight and its ability to dissociate in the temperature range of solar furnaces. However, its low density and need for cryogenic systems could impede its use, so other propellants may be preferable (such as NH_3) at the expense of lower specific impulses. The specific mission would also have to be considered in the selection of the optimal configuration and operating conditions of the solar rocket.

Appendix: Computation Description and Assumptions

Absorption Characteristics of a Mixture of Alkali Vapor and Hydrogen

In a hydrogen/alkali mixture, the light beam is predominantly absorbed by the alkali diatomic molecules (or dimers) and the absorption coefficient α_ν can be written as

$$\alpha_\nu = P_D \sigma_{\nu D} \qquad \text{(Refs. 10, 12)}$$

where $\sigma_{\nu D}$ is the absorption cross section (which has been measured by Wechsler[12] and computed by Rault[10]), and P_D is the partial pressure of alkali dimers

$$P_D = K_p P_A^2 \quad \text{with} \quad P_A = P \Big/ \left[1 + \left(\frac{1}{\alpha} - 1 \right) \frac{M_A}{M_H} \right]$$

Performance of an Alkali/Hydrogen Radiation Receiver

The gas flow configuration near the entrance of a typical radiation receiver would be fairly complex and difficult to represent analytically. However, for aperture ratios which are not too large, the radiation field within the absorber cavity is predominately axial and can be analyzed in one dimension when the side walls are assumed to be perfectly reflecting. Consequently, for the first-order analysis presented in this paper, the basic flow equations were integrated over a cross-sectional area of the absorber cavity, and a quasi-one-dimensional formulation of the energy transfer problem was made. The gas temperature profile within a cylindrical radiation receiver (Fig. 2) can be shown to be such that

$$\begin{aligned} \dot{m} C_P \frac{\mathrm{d}T}{\mathrm{d}x} &= \int_{\nu_1}^{\nu_2} \alpha_\nu (\eta_1 \eta_2 I_{\nu 0}) e^{-S_\nu(x)} \mathrm{d}\nu \\ &+ \Lambda \int_{\nu_1}^{\nu_2} \alpha_\nu \Big[-E_2(S_\nu) B_\nu(T) \\ &- \frac{1}{2} \int_0^x E_2 [S_\nu(x) - S_\nu(\tilde{x})] \frac{\mathrm{d}B_\nu}{\mathrm{d}x} \mathrm{d}\tilde{x} \\ &+ \frac{1}{2} \int_x^\infty E_2 [S_\nu(\tilde{x}) - S_\nu(x)] \frac{\mathrm{d}B_\nu}{\mathrm{d}x} \mathrm{d}\tilde{x} \Big] \mathrm{d}\nu - \pi N_u k (T - T_w) \end{aligned} \qquad \text{(A1)}$$

where T is the "mixing cup average" temperature defined as: $T = 1/\dot{m} \int \rho u T \mathrm{d}s$. In a first-order approximation, all the parameters in Eq. (A1) can also be considered to represent "mixing cup average" quantities. The first term on the right side of Eq. (A1) represents the absorption of the solar beam by the working fluid, while the second term represents the net effect of reradiation by the hot gases (emission by the hot gases at axial distance x and absorption of the reradiation emitted upstream and downstream of x, respectively). The last term represents the convection heat losses to the side walls. Equation (A1) is integrodifferential and was solved as described by Rault in Ref. 10, (diffusion method for large values of x and linearization and iteration for small values of x). The Nusselt number of a turbulent developing flow was chosen to model the convection losses.

The collection efficiency of the radiation receiver was subsequently computed as

$$\eta_3 = \dot{m} \int_{T_i}^{T_0} C_p \mathrm{d}T / \eta_1 \eta_2 \int_{\nu_1}^{\nu_2} I_{\nu 0} \mathrm{d}\nu \qquad \text{(A2)}$$

where T_0 is the maximum gas temperature which is reached when the convection heat losses to the side walls balance the enthalpy gained through radiation absorption.

Specific Impulse of an Alkali/Hydrogen Thruster

The gas velocity u_e at the exit of the thruster is such that

$$\frac{1}{2} u_e^2 = \frac{1}{2} (I_{sp} g)^2 = (1 - \alpha) \left[\int_{T_0}^{T_t} C_{pe} \mathrm{d}T + C_{pf} (T_t - T_e) \right] \qquad \text{(A3)}$$

In Eq. (A3), the contribution of the alkali metal has been neglected. The terms on the right represent, respectively, the enthalpy change as the gas flows in the converging (equilibrium flow[27]) and diverging (frozen flow) sections of the nozzle. The flow in the nozzle is assumed to be isentropic.

Thrust of an Alkali/Hydrogen Thruster

The thrust delivered by the solar rocket is $F = \dot{m}u_e = \dot{m}I_{sp}g$. For given operating conditions (stagnation temperature and pressure, alkali weight concentration), the specific impulse and the collection efficiency of the receiver are determined as shown above, and the thrust F can be written as

$$F = \eta_1 \eta_2 \eta_3 I_{sp} g \int_{\nu_1}^{\nu_2} I_{\nu 0} \mathrm{d}\nu \Big/ \int_{T_i}^{T_0} C_p \mathrm{d}T$$

To ensure that the alkali vapor entering the radiation receiver remains dry, the alkali-hydrogen mixture must be sufficiently preheated. In the absence of any auxiliary heat source, the energy available to preheat the gas mixture is the solar beam energy, which is not converted into gas enthalphy or reradiated into space. Assuming an 80% energy transfer efficiency, it can be shown that to ensure sufficient preheating, the gas mass flow rate must be such that

$$\dot{m} \int_{T_c}^{T_i} C_p \mathrm{d}T \leq 0.80\eta_1 \left(\int_0^\infty I_{\nu 0} \mathrm{d}\nu - \eta_2 \int_{\nu_1}^{\nu_2} I_{\nu 0} \mathrm{d}\nu + \pi N_u k \int (T - T_w) \mathrm{d}x \right)$$

Mission Time (Continuous Burn)

The minimum initial to final mass ratio MR of a chemical rocket designed to travel from LEO to GEO is $MR = \exp(\Delta V/u_e)$. Therefore, for a continuous burn, the mision time τ is

$$\tau = \frac{M_0}{\dot{m}} (1 - \exp(-\Delta V/u_e))$$

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